

## HEAT TRANSFER IN A CIRCULATING FLUIDIZED BED

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*On the basis of similarity theory a relationship is established for calculating the conductive-convective component of the coefficient of heat transfer in a circulating bed. Recommendations on the calculation of the total coefficient of heat transfer in a high-temperature bed are given. The gas gap between the heat-transfer surface and the first row of particles is evaluated.*

Knowledge of the regularities of heat transfer between a circulating fluidized bed (CFB) and the surfaces bounding it is important for thermal calculation of apparatuses with a CFB, foremost for furnaces burning solid highly disperse fuel.

It has been repeatedly mentioned in the literature [1-4] that heat transfer in a CFB is closely related to the conditions at the riser, where a rather complex and special structure of the two-phase flow is realized. As is known [2], in the near-wall region a descending flow of particles in two phases occur: a) differently shaped clusters of particles in which the particle concentration is close to that in the fluidized bed; b) a dilute phase where the particle concentration is relatively low and corresponds to the level of the particle concentration in the above-layer zone of the fluidized bed. Both phases move down along the walls of the vertical riser at different velocities. The general model expression for the coefficient of heat transfer has the form [2]

$$\alpha = \delta_c (\alpha_{c-c,c} + \alpha_{r,c}) + (1 - \delta_c) (\alpha_{c-c,d} + \alpha_{r,d}), \quad (1)$$

where  $\alpha_{c-c,c}$  and  $\alpha_{c-c,d}$  are the coefficients of conductive-convective heat transfer of the clusters and the dilute phase;  $\alpha_{r,c}$ ,  $\alpha_{r,d}$  are the coefficients of radiative heat transfer of these phases. Equation (1) involves five unknown functions and is hardly applicable as a basis for practical calculations, although such attempts have been made [2, 3]. To simplify (1), heat transfer can be simulated within the framework of a one-phase model assuming that the particles move at the riser walls as a uniform suspension. In this case the coefficient of heat transfer is expressed as

$$\alpha = (1/\alpha_f + 1/\alpha_s)^{-1} + \alpha_r, \quad (2)$$

where  $1/\alpha_f = kd/\lambda_f$  is the contact thermal resistance of a gas film of thickness  $\delta = kd$  separating the first row of particles from the surface;  $\alpha_s$  is the coefficient of heat transfer between the suspension and the gas film. Nearly the same scheme was realized in [4]. Equation (1) is even more simplified if we do not consider the gas layer separately:

$$\alpha = \alpha_{c-c} + \alpha_r, \quad (3)$$

where  $\alpha_{c-c}$  is the conductive-convective component of the heat transfer coefficient.

A number of publications are available that report test data on values of the coefficients  $\alpha_{c-c}$  under cold (isothermal) conditions [1, 5-10] and values of  $\alpha$  obtained on high-temperature installations burning solid-fuel particles [3, 11-14]. The main qualitative regularities are revealed:  $\alpha_{c-c}$  decreases with increase in the particle

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TABLE 1. Conditions of Experiments on Measuring the Coefficient of Heat Transfer in a Circulating Fluidized Bed

Reference	$d$ , mm	$T_{\infty}$ , °C	$L$ , m	$H$ , m	$\rho_s$ , kg/m <sup>3</sup>
[1]	0.287	25	0.4	10	2650
[5]	0.078	25	0.1	10	2857
[6]	0.046	25	0.07; 0.012	6.65	2300
[13]	0.240	850	1-10	13.5	2600
[7]	0.087	35	0.025	5.5	2650
[7]	0.227	30	0.025	5.5	2650
[8]	0.250	25	0.1	3.0	2630
[11]	0.241	343-880	1.22	7.32	3066
[11]	0.241	410-870	1.59	7.32	3066
[12]	0.188	400	1.59	7.32	2637
[12]	0.356	400	1.59	7.32	2637
[3]	0.309	200-950	0.025	5.25	2350
[3]	0.309	60-500	0.3	5.25	2350
[8]	0.170	25	0.1	3.0	2630
[14]	0.240	750-850	0.01	8.3	3000
[10]	0.125	25	0.019	11	2600
[20]	0.25	25	0.026	6.6	2470
[20]	0.5	25	0.026	6.6	1240
[20]	0.65	25	0.026	6.6	1240

diameter [2, 9, 11], with increase in the height of the location of the heat transfer surface above the gas distributor [1, 5], and with increase in the surface length [11];  $\alpha$  increases greatly with the layer temperature [2, 3, 11]. It is found that the most important quantity affecting  $\alpha$  is the mean concentration of the particles over the horizontal cross section of the riser [2-4, 11-14]. In accordance with this, the coefficient  $\alpha_{c-c}$  grew greatly with the circulating particle flow [1, 5], which governs to a great extent the distribution of the solid-phase concentration over the riser height [15].

Although the qualitative understanding of the heat-transfer process is rather good and there is a comparatively large amount of test data (see Table 1), the literature has no reliable, sufficiently universal relationships for calculating the rate of heat transfer between a CFB and the riser walls.

Our aim was to obtain simple relations for calculating coefficients of heat transfer within the framework of the one-zone model (3) using the previously developed theory of similarity between the processes of conductive-convective transfer in a CFB [15].

**Conductive-Convective Heat Transfer.** The general functional dependence of the coefficient  $\alpha_{c-c}$  on the governing parameters is the following:

$$\alpha_{c-c} = f(g, d, J_s, h, L, H, \rho_s, \rho_f, \lambda_f, c_f, u - u_t, \nu_f)^* \quad (4)$$

On the basis of the  $\pi$ -theorem of dimensionality theory [17] we write the dimensionless analog of (4)

$$\frac{\alpha_{c-c} d}{\lambda_f} = f \left( \frac{L}{d}, Ga, \frac{J_s}{\rho_s (u - u_t)}, \frac{\rho_s}{\rho_f}, \frac{(u - u_t) d}{\nu_f}, Pr, \frac{h}{d}, \frac{H}{d} \right) \quad (5)$$

\*) The excess velocity  $u - u_t$  is taken as the characteristic velocity. In our opinion, this quantity, which is actually the velocity of particle motion in the ascending loop of the riser, determines the hydrodynamic pattern of gas flow and particle motion in a CFB. Here it is pertinent to note the analogy with use of  $u - u_0$  in the description of transfer processes in a nonuniform fluidized bed [16].

The general dimensionless relation (5) is rather cumbersome and it should be simplified. Considering that under isothermal conditions the quantities  $Pr$  and  $\rho_s/\rho_f$  change comparatively little, they may be neglected in (5). We use the more common number  $Ar$  instead of  $Ga$ . The effect of the gas velocity  $u$  is taken into account only once, in the combination  $\bar{J}_s = J_s/\rho_s(u - u_f)$  (the possibility of this will be confirmed by subsequent results of generalization of test data). And, finally, instead of the two simplexes  $h/d$  and  $H/d$  we form and retain only the one simplex  $h/H$  (the simplex  $H/D$  is discarded as having no physical meaning.)\* As a result of these simplifications, (5) is reduced to the equation

$$Nu_{c-c} = f \left( \bar{J}_s, Ar, \frac{h}{H}, \frac{d}{L} \right). \quad (6)$$

We note that (6) involves the value of the dimensionless mass flow of particles  $\bar{J}_s$ . This combination, as has been shown in [15], is a generalized parameter of a CFB and reflects the similarity of hydrodynamic processes in the system.

Assuming a power-law dependence for  $f$  we obtain

$$Nu_{c-c} = A \bar{J}_s^a Ar^b \left( \frac{h}{H} \right)^c \left( \frac{d}{L} \right)^d. \quad (7)$$

The five unknown coefficients in (7) were determined using a standard procedure by processing test data (see below).

**Radiative Heat Transfer.** In a rather rarefied disperse system, which a CFB is, the radiative component of the coefficient of heat transfer between a high-temperature bed and the riser surface can constitute a substantial part [3].

The general expression for  $\alpha_r$  has the form

$$\alpha_r = \frac{\sigma_0 (T_e^2 + T_w^2) (T_e + T_w)}{1/\epsilon_e + 1/\epsilon_w - 1}, \quad (8)$$

where  $\epsilon_e$  is the effective emissivity of the CFB;  $T_e$  is the temperature of the particles at the heat transfer surface. According to the results of [18], in which the effective emissivity of particle aggregates was studied at different concentrations,  $\epsilon_e$  is determined by the following inequality

$$\epsilon_s^{0.48} < \epsilon_e < \epsilon_s^{0.31}, \quad (9)$$

where  $\epsilon_s^{0.31}$  is the emissivity of a rarefied bed, and  $\epsilon_s^{0.48}$  is the same for a dense bed. As has already been mentioned, at the heat-transfer surface a CFB consists of a cluster phase (emissivity  $\epsilon_s^{0.48}$ ) and a dilute phase (emissivity  $\epsilon_s^{0.31}$ ). For actual values of  $\epsilon_s = 0.6-0.8$  the range of variation of  $\epsilon_e$  is rather narrow according to (9), thus making it possible to suggest

$$\epsilon_e \approx (\epsilon_s^{0.48} + \epsilon_s^{0.31})/2 \quad (10)$$

for calculating the effective emissivity of a CFB.

A more intricate problem is the determination of  $T_e$ , which should, generally speaking, depend on the length of the heat-transfer surface. In fact, for motion of the particles along the riser walls their temperature

\*) We assume the presence of two independent one-dimensional spaces: a "local" one associated with the heat-transfer surface (the characteristic dimension is the length of this surface  $L$ ); a "global" one associated with the height of the riser (the characteristic dimension is the riser height  $H$ ). Within the framework of these notions the quantity  $h$  characterizes the coordinate of the "local" space in the "global" one, where the "local" space is treated as 0-dimensional. It is obvious that only the simplexes  $h/H$  and  $d/L$  are meaningful in this formulation.

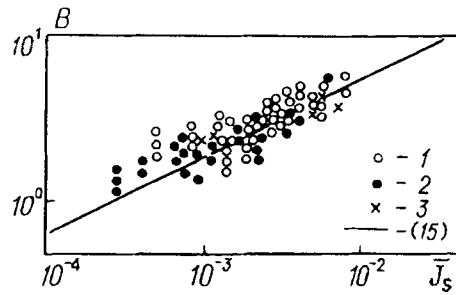


Fig. 1. Dependence of the conductive-convective component of the coefficient of heat transfer on the dimensionless circulating flow of particles: 1) [1],  $d = 0.287$  mm; 2) [5], 0.078; 3) [6], 0.046.  $B \equiv Nu_{c-c} / (h/H)^{-0.42} (d/L)^{0.43} Ar^{0.12}$ .

decreases and the effective temperature  $T_e$  in (8) is determined as the mean over the entire length of the heat exchanger:

$$T_e = \langle T_s \rangle = \frac{1}{L} \int_0^L T_s dl, \quad (11)$$

where  $T_s$  is the local temperature of the particles at the surface.

The rate of particle cooling at the surface can be evaluated from tests [19, Fig. 4-4] where the temperatures of fluidized corundum particles ( $d = 0.25-0.5, 0.5-1.0, 1.0-1.5$  mm) heated to  $940-1220^\circ\text{C}$  were measured by the photopyrometry method. We processed these data and obtained the following relationship:

$$\frac{T_s - T_w}{T_\infty - T_w} = \exp(-0.032 Fo^{0.13}). \quad (12)$$

With account for  $t = l/\nu$ , on the basis of (11) and (12) we have an expression for the mean temperature  $T_e$ :

$$T_e = T_w + (T_\infty - T_w) \frac{2.42 \cdot 10^{12}}{Fo_L} \gamma(7.962; 0.032 Fo_L^{0.13}), \quad (13)$$

where  $Fo_L = a_f L / \nu d^2$ ;  $\gamma(\alpha, x) = \int_0^x \exp(-p) p^{\alpha-1} dp$  is the incomplete gamma-function.

A calculation by (13) for the conditions  $d = 0.15 \cdot 10^{-3}$  m,  $a_f = 5.2 \cdot 10^{-4}$  m<sup>2</sup>/sec,  $\nu = 1$  m/sec [6],  $T_\infty = 1000^\circ\text{C}$ ,  $T_w = 100^\circ\text{C}$ ,  $L = 1$  m gives  $T_e \approx 915^\circ\text{C}$ , which differs from  $T_\infty$  by only 8.5%. This corresponds to an error of determination of  $\alpha_r$  by (8) in the order of 15%. In spite of some arbitrariness in this estimate and allowing for inevitably present errors of determination of  $T_w$ ,  $\epsilon_w$ , and  $\epsilon_s$ , we consider replacement of  $T_e$  by  $T_\infty$  in (8) to be allowable at this stage. Thus, the simplified formula for calculating  $\alpha_r$  has the form

$$\alpha_r = \frac{\sigma_0 (T_\infty^2 + T_w^2) (T_\infty + T_w)}{1/\epsilon_e + 1/\epsilon_w - 1}. \quad (14)$$

We note here that (14) in no way precludes the possibility of using (8) and (13).

**Generalization of Test Data.** To generalize data on  $\alpha_{c-c}$  we used (7). By the least-squares method we processed test data of [1, 5, 6], and here the dependence of  $\alpha_{c-c}$  on the length of the heat transfer surface was found preliminarily on the basis of results of [11]. The following equation was obtained:

$$Nu_{c-c} = 55 \bar{J}_s^{0.5} \left( \frac{h}{H} \right)^{-0.42} \left( \frac{d}{L} \right)^{0.43} Ar^{0.12}. \quad (15)$$

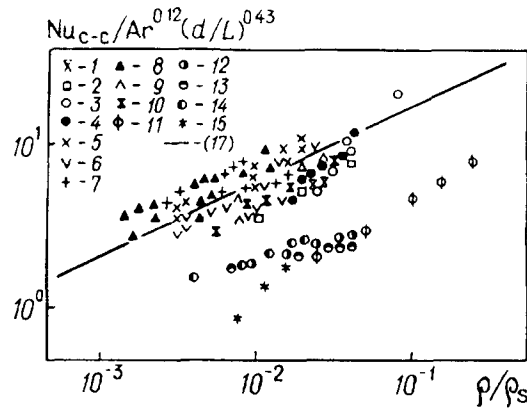


Fig. 2. Conductive-convective heat transfer in CFB (see Table 1): 1) [8],  $d = 0.250$  mm; 2) [7], 0.227; 3, 4) [12], 0.188 and 0.356; 5, 6) [11],  $L = 1.22$  and 1.59 m; 7, 8) [3], 0.025 and 0.3; 9) [7],  $d = 0.087$  mm; 10) [8], 0.170; 11) [10]; 12-14) [20],  $d = 0.50, 0.65,$  and  $0.25$  mm; 15) [14], 240.

Test points [1, 5, 6] and dependence (15) are shown in Fig. 1. The mean-root-square deviation of the test points from those calculated is 17%. The range of verification of (15) is  $5.91 \leq J_s \leq 68.7 \text{ kg/m}^2 \cdot \text{sec}$ ,  $6.65 \leq H \leq 10$  m,  $0.046 \leq d \leq 0.287$  mm,  $0.07 \leq L \leq 0.4$  m.

The obtained correlation (15) makes it possible to find the functional dependence of  $\alpha_{c-c}$  on the mean particle concentration over the horizontal cross section of the riser. In [15] we found a simple equation for determining  $\rho$ :

$$\frac{\rho}{\rho_s} = \bar{J}_s \left( \frac{h}{H} \right)^{-0.82} \quad (16)$$

With account for (16) it is easy to obtain from (15) the sought relation between  $\alpha_{c-c}$  and  $\rho$ :

$$\text{Nu}_{c-c} = 55 \left( \frac{\rho}{\rho_s} \right)^{0.5} \left( \frac{d}{L} \right)^{0.43} \text{Ar}^{0.12} \quad (17)$$

A rather large amount of data on the dependence of the coefficient of heat transfer on the particle concentration  $\rho$  is known from the literature. The results of a comparison of test data [3, 7, 8, 10-12, 14] \*) with those found by (17) presented in Fig. 2 show rather satisfactory agreement within a wide range of variation of experimental conditions (see Table 1).

It is important to note that test points [10, 14, 20] obtained for small lengths of the heat transfer surface, as is seen from Fig. 2, fall outside the common field of experimental points and form an individual group. This indicates that (15) and (17) are inapplicable for describing local heat transfer (heat transfer with surfaces of small length). This limiting case is very interesting from the viewpoint of understanding the mechanism of heat transfer and should be considered in more detail.

As is known [21], for very small times of contact between the disperse medium and the heat transfer surface (this corresponds to the case of small  $L$ ) the main thermal resistance is concentrated in a gas film of thickness  $\delta \sim d$  separating the particles of the first row from the wall. In accordance with this, expression (2) is simplified and reduced to the form

$$\alpha = \alpha_f + \alpha_r, \quad (18)$$

\*) In the case of a high-temperature CFB the test data on the quantities  $\alpha$  were recalculated and  $\alpha_{c-c}$  was found by the formula  $\alpha_{c-c} = \alpha - \alpha_r$ , where  $\alpha_r$  was calculated by (14).

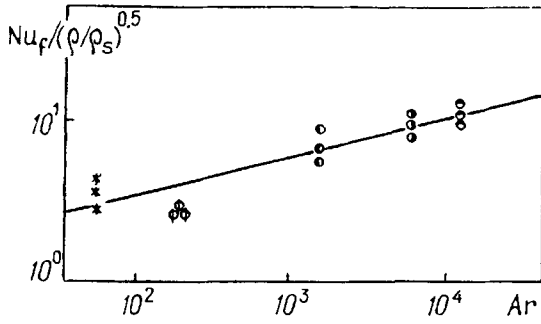


Fig. 3. Limiting values of the conductive-convective component of heat transfer in a CFB (for the notation see Fig. 2).

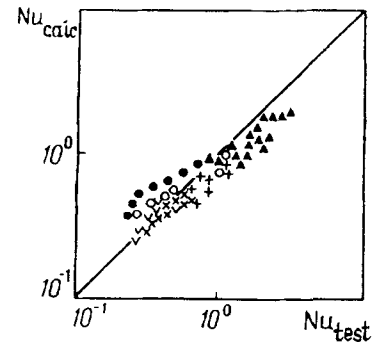


Fig. 4. Combined heat transfer in a CFB (for the notation see Fig. 2).

where  $\alpha_f$  is the maximum (limiting) value of the conductive-convective component of heat transfer in the CFB. The mentioned limiting values of  $\alpha_{c-c}$ , where the dependence on  $L$  disappears, were generalized separately by the relation

$$Nu_f = 1.23 \left( \frac{\rho}{\rho_s} \right)^{0.5} Ar^{0.23}, \quad (19)$$

which is shown in Fig. 3 in combination with test points [10, 14, 20]. The obtained correlation (19) allows one to evaluate the gas gap  $\delta = kd$  between the riser wall and the first row of particles. Since  $\alpha_f = \lambda_f/\delta$ , we obtain  $Nu_f = 1/k$  for  $Nu_f = \alpha_f d/\lambda_f$ . Finally, for evaluation of the scale coefficient  $k$ , from (19) we have

$$k = 0.81 \left( \frac{\rho}{\rho_s} \right)^{-0.5} Ar^{-0.23}. \quad (20)$$

The test values of  $Nu_f$  shown in Fig. 3 give the following estimate for  $k$ :  $1 \leq k \leq 15$ . The range of variation of  $Ar$  is 51–12,360, with greater  $k$  corresponding to smaller  $Ar$ .

To determine more accurately the range of applicability of (15) and (17) we should find an estimate of the limiting values of  $L$ , where the dependence of  $\alpha_{c-c}$  on  $L$  can be neglected and formula (19) can be used. In [21], on the basis of an analysis of unsteady heat transfer of an infiltrated disperse medium with the surface, we found that for small contact times ( $Fo = a_f t/d^2 < 5$ ) the heat transfer virtually ceases to depend on  $Fo$ , thus reaching the limiting values  $\alpha_f = \lambda_f/\delta$ . With this result at hand we can easily obtain the sought estimate

$$L \approx 10^5 d^2, \quad (21)$$

where  $L$  and  $d$  are in meters. We note that to obtain (21) we used  $t = L/\nu$ ,  $\nu \approx 1$  m/sec [6],  $a_f = 2.25 \cdot 10^{-5}$  m<sup>2</sup>/sec.

Figure 4 presents results of a comparison of test data on the total coefficient of heat transfer of a high-temperature CFB [3, 11, 12] with those calculated by the equation

$$Nu = 55 \left( \frac{\rho}{\rho_s} \right)^{0.5} \left( \frac{d}{L} \right)^{0.43} Ar^{0.12} + \frac{\sigma_0 d (T_\infty^2 + T_w^2) (T_\infty + T_w)}{\lambda_f (1/\epsilon_c + 1/\epsilon_w - 1)}, \quad (22)$$

which follows from (3), (14), and (17). As is seen, formula (22) describes quite satisfactorily (the mean-root-square deviation of the test points from the calculated ones is 21%) the regularities of combined heat transfer in a CFB within a rather wide range of variation of the test conditions:  $343 \leq T_\infty \leq 880^\circ\text{C}$ ;  $0.188 \leq d \leq 0.356$  mm;  $0.025 \leq L \leq 1.59$  m;  $5.25 \leq H \leq 7.32$  m.

Correlation (22) reflects the effect of the main factors on the rate of heat transfer in a CFB and can be recommended, in combination with (15), for use in engineering practice.

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## NOTATION

$a_f$ , thermal diffusivity of the gas;  $Ar = gd^3[(\rho_s/\rho_f) - 1]/\nu_f^2$ , Archimedes number;  $c$ , specific heat capacity;  $d$ , particle diameter;  $Fo = a_f t/d^2$ , Fourier number;  $g$ , acceleration of gravity;  $Ga = gd^3/\nu_f^2$ , Galileo number;  $h$ , height above the gas-distributing grid;  $H$ , riser height;  $J_s$ , mass circulating flow of the particles;  $\bar{J}_s = J_s/\rho_s(u - u_1)$ , dimensionless circulating flow of particles;  $L$ , length of the heat-transfer surface;  $k$ , scale coefficient determining the gas-film thickness at the heat-transfer surface;  $Nu = \alpha d/\lambda_f$ , Nusselt number;  $Pr = c_p \rho_f \nu_f/\lambda_f$ , Prandtl number;  $T$ , temperature;  $t$ , time of residence of a particle aggregate at the heat-transfer surface;  $u$ ,  $u_0$ ,  $u_1$ , velocity of gas filtration, velocity of onset of fluidization, and velocity of floating, respectively;  $v$ , velocity of the descending flow of particles at the riser walls;  $\alpha$ , coefficient of heat transfer;  $\delta_c$ , portion of the heat-transfer surface adjacent to clusters;  $\epsilon$ , emissivity;  $\nu_f$ , kinematic viscosity of the gas;  $\rho$ , mean bed density (particle concentration) over the horizontal cross section of the riser;  $\sigma_0$ , Stefan-Boltzmann constant;  $\lambda_f$ , thermal conductivity of the gas. Subscripts: c, cluster phase; c-c, conductive-convective; d, dilute phase; e, effective; f, fluid (gas); r, radiative; s, particles; w, heat-transfer surface;  $\infty$ , bed core; t, conditions of floating of a single particle.

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